Negative Matter Propulsion

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Negative matter is a hypothetical form of matter whose active-gravitational, passive-gravitational, inertial, and rest masses are opposite in sign to normal, positive matter. Negative matter is not antimatter, which as far as is known has normal (positive) mass. If an object made of negative matter could be obtained and coupled by elastic, gravitational, or electromagnetic forces to an object containing an equal amount of positive matter, the interactions between the two objects would result in an unlimited amount of unidirectional acceleration of the combination without the requirement for an energy source or reaction mass. In this paper, it is shown in exhaustive detail that, despite their unbelievable propulsive capabilities, negative matter propulsion systems do not violate the Newtonian laws of conservation of linear momentum and energy. Thus, logical objections to the existence of negative matter must be found elsewhere than in Newtonian mechanics. Suggestions are made where evidence for the existence of negative matter might be found.

Background

FOR over two decades, I have been giving popular science lectures on anomalous concepts in physics,[1] expecially those that might be useful in constructing science fiction stories. One concept that usually draws an amazed response from the audience is the concept of negative matter propulsion. The first scientific paper discussing this anomalous physical concept was published by Bondi.[2]

To quickly get to the crux of the problem, let me repeat here the material I usually give in my talks. As will be explicitly shown later, an object with negative mass gravitationally repels all other types of matter, both positive and negative, whereas a positive mass attracts all other types of matter, including negative matter. Thus, as shown in Fig. 1, if we take a ball of negative matter of mass $-M$ and place it near a rocket ship of mass $+M$ of equal magnitude, the gravitational field of the negative mass will repel the positive mass, while the gravitational field of the positive mass will attract the negative mass, and the rocket ship and the ball of negative matter will go off in the same direction with an acceleration proportional to the gravity force between them. This truly miraculous negative matter propulsion system seems to provide an unlimited amount of unidirectional acceleration without requiring either an energy source or reaction mass.

Now at first glance, it would seem that this result proves negative matter cannot exist. The system starts out with two objects standing still, then after awhile, the two objects are moving off to the right with nothing going in the opposite direction. This obviously violates the laws of conservation of linear momentum and energy.

Amazingly enough, it does not.

When two objects are at zero velocity, the total momentum of the system is zero. After the two objects have reached velocity v, their combined momentum ΣP is still zero

$$\Sigma P = P_+ + P_- = (+M)v + (-M)v = 0 \qquad (1)$$

because the ball made of negative matter has negative momentum.

There is also no violation of the law of conservation of energy. When the two objects are at zero velocity, the total energy of the system is zero. After the two objects have reached v, their combined kinetic energy ΣP is still zero (I multiplied the equation by 2 to simplify its form)

$$2\Sigma E = 2E_+ + 2E_- = (+M)v^2 + (-M)v^2 = 0 \qquad (2)$$

because the ball of negative matter has negative kinetic energy.

In addition, it requires no energy to make the marvelous negative matter ball to run our miraculous negative matter propulsion system. As long as we generate positive and negative matter in equal amounts during our fabrication process, the total energy needed to provide the rest mass for the combined system is zero

$$(+M)c^2 + (-M)c^2 = 0 \qquad (3)$$

because the negative matter ball has negative rest mass energy. (At this point, the audience usually breaks into incredulous guffaws, or at least stifled giggles.)

Although negative rest mass energy, negative kinetic energy, and negative matter are not standard concepts in Newtonian mechanics, they do not immediately seem to lead to obvious logical contradictions. At the very least, they have enough logical validity that they can be used in writing science fiction,[1] or in speculating on possible future "breakthroughs" in advanced propulsion physics.[3]

Recently, I presented this concept once again at a popular lecture for novice science fiction writers. The next speaker, the famous science fiction author, Larry Niven, was in the audience. During the question and answer session, he raised a question that seemed to lead to a logical contradiction that

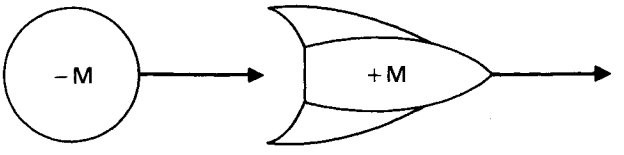

Fig. 1 Negative matter gravitational propulsion.

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proved negative matter was logically forbidden. Niven's question was essentially this:

> Suppose the positive mass $+M_+$ is greater in magnitude than the negative mass $-M_-$ and the two are connected together by a stiff rod. The acceleration would, of course, be somewhat different than the case in which the masses were equal in magnitude, but if they did reach v, since the positive mass is greater than the negative mass, a calculation of the total momentum would not produce zero
>
> $$\Sigma P = P_+ + P_- = (+M_+)v + (-M_-)v$$
> $$= (M_+ - M_-)v \neq 0 \qquad (4)$$
>
> In that case, would not the law of conservation of momentum be violated?

The rest of this long, tedious, and admittedly incomplete paper is the first step in an attempt to answer that simple question.

Introduction

I define negative matter as a hypothetical form of matter whose mass is opposite in sign to normal positive matter. Negative matter is not antimatter. Antimatter exists and is known to have a positive inertial mass. Its gravitational mass is unknown, although there are a number of experiments in progress to measure the passive gravitational mass of an antiproton or an antihydrogen atom.[4] It is expected that antimatter will be found to have a positive gravitational mass, since otherwise, it would be possible to build a perpetual motion machine.[5]

The first modern scientific paper to discuss negative matter was that by Bondi,[2] who offered a general discussion of the concept of negative mass in terms of Einstein's theory of gravity, the General Theory of Relativity. Bondi showed that the concept of a negative mass chasing a positive mass with constantly increasing velocity is consistent with general relativity. Bondi's work was later expanded by Bonnor and Swaminarayan,[6] who found an exact solution to the complete Einstein field equations that accurately describe a uniformly accelerated pair of gravitationally interacting bodies with opposite masses, as shown in Fig. 1. The only difference between the obvious Newtonian solution and the general relativity solution is that in the general relativity solution, the two masses are not quite equal and opposite because they are being measured in an accelerating reference frame. For the two masses to keep a constant separation in the accelerated reference frame, the negative mass should be very slightly larger than the positive mass, depending on the initial separation.

Since negative matter is not logically forbidden by general relativity, the experimental fact of its nonexistence in the observed universe must be due to some other theory of physics. In this paper, I explore whether the existence of negative matter leads to any logical contradictions in Newtonian mechanics. I find that the existence of negative matter would produce some startling and potentially highly useful results, but this startling behavior does not violate any Newtonian conservation laws and does not lead to any logical contradictions. If negative matter is logically forbidden, the proof must come from some other theory of physics rather than simple Newtonian mechanics. The nonexistence proof could be found in the special theory of relativity, quantum mechanics, the various elementary particle theories, or the yet-to-be-invented unified field theory.

Because I am limiting this paper to an evaluation of negative matter in Newtonian mechanics, I will make a number of simplifying assumptions in the following pages. The reader is invited to explore the ramifications of those simplifying assumptions in an effort to find if they contain the clue as to why negative matter logically cannot exist.

The first assumption I will make is that all objects are point particles. (I will ignore the internal structure of the objects.) This assumption is one of the weak spots of this paper. Since negative matter gravitationally repels all other types of matter, a collection of negative matter particles would not hold together through self-gravitational forces. Also, since negative matter responds to a force by moving in a direction opposite to the direction of the force vector, normal molecular binding forces will not hold a collection of negative matter particles together unless they have a very special force law. I will not speculate further on the internal structure of the negative matter objects, but simply assume they are massive point particles.

The second assumption I will make is that all velocities are subrelativistic. The third assumption is that all forces are linear, in that the forces are additive. The fourth assumption is that Newtonian mechanics can be sufficiently explored by using only two particles with their dynamical behavior limited to one dimension. By using three or more objects, or allowing dynamics to occur in two or three dimensions where the law of conservation of angular momentum comes into play, it may be possible to bring out a logical inconsistency that will prove the nonexistence of negative matter, but I doubt it. I will not attempt those analyses in this paper; it is far too long already.

I first conduct, in tedious detail, the analysis for the case of two particles connected by a spring. I start with the case in which the two particles are made of positive matter, then I carry out a comparative analysis in which one of the particles is made of negative matter. I then complete similar analyses in which the coupling between the particles is gravitational and electrostatic. The results are essentially the same, so at the request of the reviewers and editor, these analyses have been dropped from this version of my original paper. Readers interested in those details can read the longer preprint version of the paper.[7]

I show that for each different type of force law used to couple the positive matter particle to the negative matter particle, neither the law of conservation of linear momentum nor the law of conservation of total energy is violated, even if the masses of the two objects do not have the same magnitude, thus partially answering Niven's question.

Definition of Mass

There are three operationally distinct definitions of mass possible in Newtonian mechanics.[2] They are active gravitational mass, passive gravitational mass, and inertial mass.

The active gravitational mass M_a of a first particle produces a gravitational scalar potential ϕ that is linearly proportional to M_a and inversely proportional to the distance r from the position of the particle to the field point

$$\phi = -GM_a/r \qquad (5)$$

where $G = 6.67 \times 10^{-11}$ m^3/kg·s^2 is the Newtonian gravitational constant. The zero point for the potential is at $r \to \infty$, the the negative sign indicates an attractive potential of one positive mass for another positive mass.

The gravitational acceleration field A is then just the negative of the gradient of the gravitational potential

$$A = -\text{grad}\phi = -GM_a/r^2 \qquad (6)$$

The passive gravitational mass M_p of a second particle at the field point r from the first particle responds to A generated by the first particle to produce a force F on the second particle that is linearly proportional to the passive gravitational mass of the second particle and the gravitational acceleration field produced by the first particle

$$F = M_pA = -GM_pM_a/r^2 \qquad (7)$$

The inertial mass M_i of the second particle then responds to F by producing an acceleration a that is linearly proportional to and in the direction of the applied force and inversely proportional to M_i:

$$a = F/M_i = -GM_pM_a/M_ir^2 \tag{8}$$

In addition, Einstein's Special Theory of Relativity allows for an additional operational definition of mass, the rest mass M_0. The M_0 of an object produces, upon conversion of M_0 to energy, an amount of energy given by

$$E = M_0c^2 \tag{9}$$

where $c = 300$ Mm/s is the speed of light.

For normal positive matter, all the masses are equal and positive, or $M_a = M_p = M_i = M_0 > 0$. For this paper, I define negative matter as matter in which all the different definitions of masses are equal and negative, or $M_a = M_p = M_i = M_0 < 0$.

Center of Mass

The concept of the center of mass of a collection of massive point particles is very useful if the particles have positive mass. Let us assume we have two particles of positive mass $+M_1$ and $+M_2$ at the points x_1 and x_2, as shown in Fig. 2.

The center of mass X of a system of two particles is defined as

$$X = \frac{+M_1}{M_1+M_2}x_1 + \frac{+M_2}{M_1+M_2}x_2 = \frac{M_1x_1+M_2x_2}{\Sigma M} \tag{10}$$

where I have defined the summed total mass of the system as $\Sigma M = M_1 + M_2$ to make future equations simpler. The center of mass for the two positive matter particles shown in Fig. 2a is always between the two particles and is closer to the more massive particle. The center of mass does not shift as forces generated by one particle affect the other particle. Thus, a typical mathematical technique in ordinary, "positive" Newtonian mechanics is to shift to a center-of-mass coordinate system and carry out calculations from there. This change in coordinates to a center-of-mass system is not always possible when one of the particles is made of negative matter.

Suppose that one of the particles in Fig. 2a is made of negative matter. Specifically, as shown in Fig. 2b, let us assume that $M_1 = +M_+$ and $M_2 = -M_-$, where both M_+ and M_- are positive "magnitudes" of the masses of the particles.

We will now carry out the same procedure as before to calculate X. Substituting these definitions into Eq. (10) produces

$$X = \frac{+M_+}{M_+-M_-}x_1 + \frac{-M_-}{M_+-M_-}x_2 = \frac{M_+x_1-M_-x_2}{\Delta M} \tag{11}$$

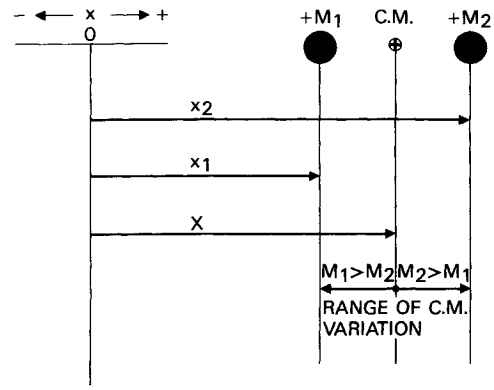
where again, to make future equations simpler, I have defined the net total mass of the system as $\Delta M = M_+ - M_-$.

For the case in which the magnitude of the mass of the positive matter particle is greater than the magnitude of the mass of the negative matter particle, the net total mass of the system is positive, or for $M_+ > M_-$, $\Delta M = M_+ - M_- > 0$. If we substitute the relation $M_+ = M_- + \Delta M$ into Eq. (11) for X, we get

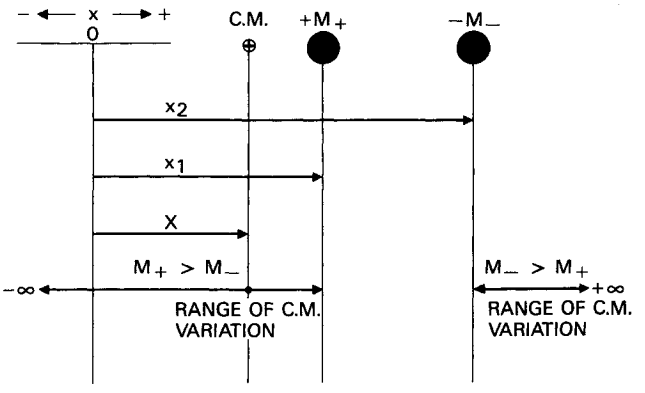
$$X = x_1 - (M_-/\Delta M)(x_2-x_1) \tag{12}$$

Since ΔM, (x_2-x_1), and M_- are all positive quantitites if $M_+ > M_-$, this means that X is outside the two points in the $-x$ direction from the positive matter particle at x_1, as shown in Fig. 2b. As M_+ becomes very much larger than M_-, X approaches x_1. As M_+ approaches M_-, X moves off in the $-x$ direction in Fig. 2b toward the point $-\infty$.

For the case in which the magnitude of the mass of the positive matter particle is less than the magnitude of the mass



a) Two positive mass particles



b) Two opposite mass particles

Fig. 2 Center of mass of two point particles.

of the negative matter particle, the net total mass of the system is negative, or for $M_+ < M_-$, $\Delta M = M_+ - M_- = -(M_+ - M_-) < 0$.

If we substitute the relation $M_- = M_+ - \Delta M$ into Eq. (11) for X, we get

$$X = x_2 - (M_+/\Delta M)(x_2-x_1) = x_2 + (M_+/\Lambda M)(x_2-x_1) \tag{13}$$

where I have defined $\Lambda M = |\Delta M| = M_- - M_+$. Since ΛM, (x_2-x_1), and M_- are all positive quantities if $M_+ < M_-$, this means that X is outside the two points in the $+x$ direction from the negative mass point at x_2, as shown in Fig. 2b. As M_- becomes much larger than M_+, X approaches x_2. As M_- approaches M_+, X moves off in the $+x$ direction toward the point $+\infty$, as shown in Fig. 2b. From this, we see that when the positive and negative matter particles have equal magnitudes, the center of mass is at $\pm\infty$. No wonder strange things can happen.

Elastic Coupling of Two Point Particles

I will now assume that I have two point particles coupled by a perfect spring. This is a more realistic version of the "stiff rod," which Niven had assumed could be attached between the two opposite matter objects to cause them to move at the same velocity. Although the original question concerned two opposite matter objects interacting through gravitational forces with a stiff rod between them to hold them at constant separation, the stiff rod also applies forces to the objects. As we shall see, it is the elasticity of a realistic nonrigid rod that keeps the conservation laws from being violated.

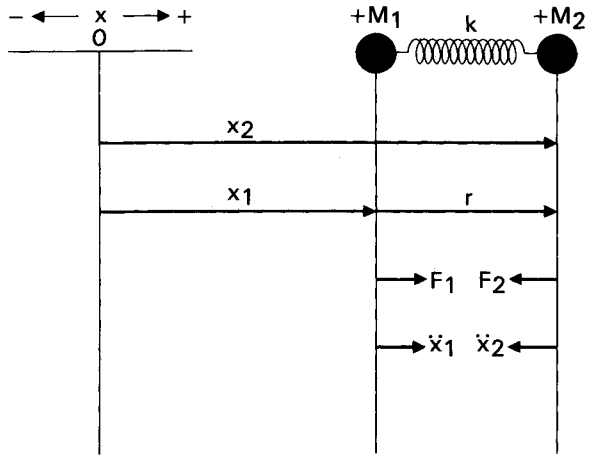
Negative Matter Propulsion Using Springs

It turns out that since the force we can apply to a negative matter object with a strong, stiff spring is much greater than the force we can apply using gravitational or electrostatic fields, a simple spring coupling gives us a highly effective and easily controlled propulsion system that can take us anywhere—at any acceleration we can stand. To make the "ideal" propulsion system, we just need a negative matter particle in the "engine" room that has a negative mass that equals in magnitude the positive mass of the entire ship. If we want to go forward, we pull a spring from the back wall of the engine room and hook it to the negative matter particle. Immediately, the perverse inertial reaction of the negative matter particle will cause it to accelerate off in the forward direction, pulling the spacecraft forward with an acceleration that is proportional to the strength of the spring. To stop accelerating, merely unhook the spring. To decelerate the ship to a stop, replace the spring coming from the back wall of the engine room with a spring coming from the forward wall. Simple, isn't it?

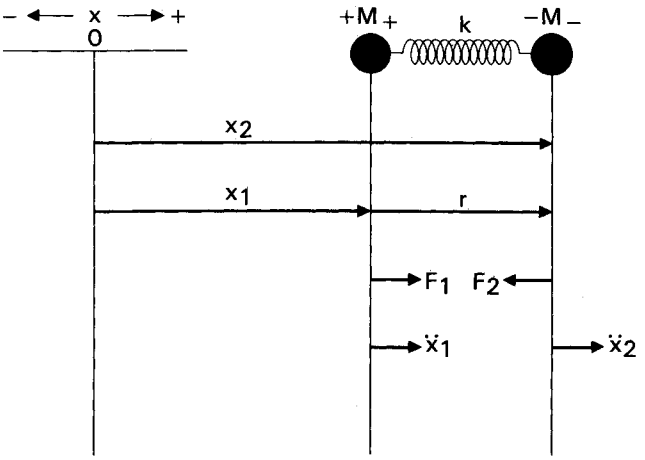
Preposterous, you say? I will now show in detail that this negative matter propulsion system does not violate the classical laws of conservation of energy and linear momentum, even if the magnitudes of the masses are not the same.

Two Particles of Positive Matter

The analysis starts out with a brief calculation of the well-known behavior of two positive matter particles connected by a spring. The calculations are then repeated in detail for two particles with masses of opposite sign. As shown in Fig. 3a, I first assume that we have two point particles at positions x_1 and x_2, with positive masses $+M_1$ and $+M_2$, connected by a spring with an elastic constant k. To make the form of the equations simpler, I will assume the spring is massless and has zero initial length, so that the force exerted by one end of the spring is directly proportional to the length of the spring, or $F=-kr$. Since the spring is connected to the two point particles, the length of the spring r is equal to the separation between the two point particles, or $r=x_1-x_2$.



a) Two positive mass particles



b) Two opposite mass particles

Fig. 3 Particles coupled by elastic forces.

The stretched spring exerts a force on each point object given by

$$F_1 = +kr \quad (\text{force in } +x_1 \text{ direction on } M_1) \tag{14}$$

$$F_2 = -kr \quad (\text{force in } -x_2 \text{ direction on } M_2) \tag{15}$$

The forces induced by the stretched spring on the two point particles causes them to respond with accelerations in the direction of the force that are inversely proportional to the (positive) inertial masses of the two objects

$$\ddot{x}_1 = \frac{F_1}{+M_1} = +\frac{k}{M_1}r \tag{16}$$

$$\ddot{x}_2 = \frac{F_2}{+M_2} = -\frac{k}{M_2}r \tag{17}$$

These are the two equations of motion to be solved. The solutions are well known as

$$x_1 = X_0 - \frac{kr_0}{M_1\Omega^2}\cos\Omega t = X_0 - \frac{M_2 r_0}{\Sigma M}\cos\Omega t \tag{18}$$

$$x_2 = X_0 + \frac{kr_0}{M_2\Omega^2}\cos\Omega t = X_0 + \frac{M_1 r_0}{\Sigma M}\cos\Omega t \tag{19}$$

where Ω is the angular frequency of oscillation of the system given by

$$\Omega^2 = k\Sigma M/M_1M_2 \tag{20}$$

and X_0 is the center of mass of the system defined by Eq. (10). The two point particles oscillate sinusoidally back and forth around the center of mass at Ω, each with an amplitude that is inversely proportional to its inertial mass. There should be no doubt that this system of two positive matter particles obeys the Newtonian laws of conservation of linear momentum and energy, but those who have doubts will find the proof in the longer preprint version of this paper.[7]

Two Particles of Opposite Matter

I will consider the case shown in Fig. 3b where there are two particles made of opposite types of matter. Particle M_1 will be made of positive matter with a mass $+M_+$, whereas particle M_2 will be made of negative matter with a mass $-M_-$. With this definition, M_+ and M_- are the magnitudes of the two masses and are positive as defined. M_+ is not necessarily equal to M_-. In Fig. 3b, the tension force exerted by each end of the spring is the same as in the two positive-matter particles case, and the forces on the two particles are defined as before by Eqs. (14) and (15). The acceleration response of the particles is different for the two opposite-matter particles case, since by comparing Figs. 3a and 3b, we see that the acceleration response of the particle made with negative matter is opposite in direction because of the negative inertial mass of the particle. The two equations of motion are

$$\ddot{x}_1 = \frac{F_1}{M_1} = \frac{+kr}{+M_+} = \frac{kr}{M_+} \tag{21}$$

$$\ddot{x}_2 = \frac{F_1}{M_2} = \frac{-kr}{-M_-} = \frac{kr}{M_-} \tag{22}$$

Because of the perverse behavior of the negative matter particle, both masses accelerate off in the same direction. To solve these equations of motion, we first form an equation describing the second time derivative of r by subtracting Eq. (21) from (22)

$$\ddot{r} = \ddot{x}_2 - \ddot{x} = \left(\frac{1}{M_-} - \frac{1}{M_+}\right)kr = \left(\frac{\Delta M}{M_+ M_-}\right)kr \tag{23}$$

where $\Delta M = M_+ - M_-$. The solutions to this equation of motion for the length of the spring depend on the relative magnitudes of the positive mass and negative mass.

Equal Mass Magnitude Case

If the magnitude of mass of the positive matter particle is equal to that of the negative matter particle, then $M_+ = M_-$ and $\Delta M = 0$. In this case, the equation of motion for the length of the spring is extremely simple, since Eq. (23) reduces to

$$\ddot{r} = 0 \tag{24}$$

This means that the length of the spring remains at its initial length at all times t

$$r = r_0 \tag{25}$$

Substituting Eq. (25) into Eqs. (21) and (22) produces the expected but bizarre result that both the positive and negative matter particles move off in the positive x direction at a constant a that is proportional to the strength of the spring k, the initial extension of the spring r_0, and inversely proportional to the magnitudes (equal) of the two masses

$$\ddot{x}_1 = \ddot{x}_2 = \frac{kr_0}{M_+} = \frac{kr_0}{M_-} = a \tag{26}$$

Since the acceleration of each particle is constant, the solution is well known, and the velocity of each particle as a function of time is

$$\dot{x}_1 = \dot{x}_2 = at \tag{27}$$

We will now check to make sure that the conservation of momentum is not being violated. At the initial time $t = 0$, the velocities of both the particles were zero, so the total momentum of the system was $\Sigma P = 0$. At the later time, the total momentum of the system is

$$\Sigma P = M_1\dot{x}_1 + M_2\dot{x}_2 = M_+ at - M_- at = 0 \tag{28}$$

since $M_+ = M_-$. Thus, the total linear momentum of the system remains zero, even though both particles are moving off in the positive x direction at constantly increasing velocity. Because the magnitudes of the masses of the two particles are equal, the increased positive linear momentum gained by the positive matter particle is compensated by the increased negative linear momentum gained by the negative matter particle.

In the same manner, the law of conservation of energy also is not violated. Note that the initial kinetic energy of the two point particles is zero, but there is potential energy stored in the spring given by

$$2E_s = kr^2 \qquad (= kr_0^2 \quad \text{for} \quad t = 0) \tag{29}$$

The total system energy at any t after that is then

$$\begin{aligned} 2\Sigma E &= M_1\dot{x}_1^2 + M_2\dot{x}_2^2 + kr^2 \\ &= M_+(at)^2 - M_-(at)^2 + kr_0^2 = kr_0^2 \end{aligned} \tag{30}$$

showing that the total energy of the system remains constant, since the increase in positive kinetic energy of the positive matter particle is compensated by the increase in negative kinetic energy of the negative matter particle.

Negative Mass Magnitude Larger

If the magnitude of mass of the negative matter particle is larger than that of the positive matter particle, then the equation of motion for the length of the spring changes. For $M_- > M_+$, $\Delta M = M_+ - M_- < 0$, and Eq. (23) becomes

$$\ddot{r} = -(k\Lambda M/M_+ M_-)r = -\Omega^2 r \tag{31}$$

where, to make the equations simple, I have defined Ω by

$$\Omega^2 = k\Lambda M/M_+ M_- \tag{32}$$

where $\Lambda M = M_- - M_+$ is a positive quantity.

The solution for the negative mass magnitude larger case is similar to the solution of the two positive matter particles case, except that the frequency of oscillation of the spring now depends on the magnitude of the mass difference ΛM. Note that for the case in which the magnitude of mass of the negative matter particle is much larger than that of the positive matter particle, $\Lambda M \to M_-$, and Eq. (32) becomes

$$\Omega^2 \to k/M_+ \tag{33}$$

This is the same frequency of oscillation that would be obtained if the larger mass were made of positive matter and were acting as a large, nearly motionless, inertial body, while the smaller mass oscillated back and forth at the end of the spring. Thus, from an inertial viewpoint, a large negative mass behaves like a large positive mass.

The solution of the equation of motion (31) for the length of the spring is well known:

$$r = r_0 \cos\Omega t \tag{34}$$

Substituting this equation into the equation of motion for the individual particles [Eqs. (21) and (22)], we get

$$\ddot{x}_1 = (kr_0/M_+)\cos\Omega t \tag{35}$$

$$\ddot{x}_2 = (kr_0/M_-)\cos\Omega t \tag{36}$$

which have the well-known solutions

$$x_1 = X_0 - (M_- r_0/\Lambda M)\cos\Omega t \tag{37}$$

$$x_2 = X_0 - (M_+ r_0/\Lambda M)\cos\Omega t \tag{38}$$

where

$$X_0 = \frac{M_- x_{20} - M_+ x_{10}}{\Lambda M} \tag{39}$$

Equations (37) and (38) look similar to the solutions given in Eqs. (18) and (19) for motion in the two positive-matter particles case, but there are significant differences. First, X_0 is not between the two masses, but at some point to the outside of the negative mass in the $+x$ direction (Fig. 2b). Second, the two particles oscillate in the same direction. The magnitude of oscillation of the positive matter particle given by x_1 in Eq. (37) is larger than the magnitude of oscillation of the negative matter particle given by x_2 in Eq. (38), since the oscillation amplitude for each particle depends on the magnitude of the mass of the other particle. At $t = 0$, the particles are at their maximum negative x extension from the X_0, and as time proceeds, they move in the $+x$ direction together, the negative matter particle leading the positive matter particle. The positive matter particle, moving faster, catches up with the negative matter particle, decreasing the distance between them. The

length of the spring decreases, the spring force decreases, and the response of the two particles decreases. The two point masses switch places at midoscillation, the spring force reverses, and the particles are decelerated until the end of the oscillation is reached and the particles start moving in the opposite direction.

Note that as the magnitude of the two masses approaches each other and $\Delta M \to 0$, the frequency of oscillation given by Eq. (32) approaches zero, the magnitude of the oscillation of the two particles given by Eqs. (37) and (38) approaches infinity, and the behavior of the system approaches the unidirectional, constant acceleration case in which the two masses are both equal and opposite.

The velocity of the individual particles is obtained by taking the time derivative of Eqs. (37) and (38) to obtain

$$\dot{x}_1 = \frac{\Omega M_- r_0}{\Delta M} \sin\Omega t = \frac{kr_0}{M_+ \Omega} \sin\Omega t \tag{40}$$

$$\dot{x}_2 = \frac{\Omega M_+ r_0}{\Delta M} \sin\Omega t = \frac{kr_0}{M_- \Omega} \sin\Omega t \tag{41}$$

Now that we have the velocities of the two point particles at any time, we can check to see that the system obeys the laws of conservation of momentum and energy. At $t = 0$, the initial velocities of the particles were zero, so the total linear momentum of the system was zero. At any time after that, the linear momentum of the system is

$$\Sigma P = M_+ \dot{x}_1 - M_- \dot{x}_2 = \frac{kr_0}{\Omega} \sin\Omega t - \frac{kr_0}{\Omega} \sin\Omega t = 0 \tag{42}$$

The law of conservation of linear momentum is therefore not violated.

In order to check on the law of conservation of energy, we must remember that the initial energy of the system is just the energy in the spring as given by Eq. (29), since the kinetic energy of the particles at $t = 0$ is zero. The total energy at any time later is

$$\begin{aligned} 2\Sigma E &= M_1 \dot{x}_1^2 + M_2 \dot{x}_2^2 + kr^2 \\ &= M_+ \frac{\Omega^2 M_-^2 r_0^2}{\Delta M^2} \sin^2\Omega t + M_- \frac{\Omega^2 M_+^2 r_0^2}{\Delta M^2} \sin^2\Omega t \\ &\quad + kr_0^2 \cos^2\Omega t \\ &= kr_0^2 \end{aligned} \tag{43}$$

where the last step was made by substituting in Eq. (32) for Ω and reducing. The law of conservation of total energy is therefore not violated.

Positive Mass Magnitude Larger

If the magnitude of the mass of the positive matter particle is larger than that of the negative matter particle, or $M_+ > M_-$, then $\Delta M = M_+ - M_- > 0$, and Eq. (23) becomes

$$\ddot{r} = r/T^2 \tag{44}$$

where I have defined the time constant T by the equation

$$1/T^2 = k\Delta M/M_+ M_- \tag{45}$$

The equation of motion (44) has the general solution

$$r(t) = (r_0/2)(e^{t/T} + e^{-t/T}) \tag{46}$$

and the time derivatives

$$\dot{r} = (r_0/2T)(e^{t/T} - e^{-t/T}) \tag{47}$$

$$\ddot{r} = (r_0/2T^2)(e^{t/T} + e^{-t/T}) = r/T^2 \tag{48}$$

We see from Eq. (46) that after an initial transient that involves both an exponential growth and exponential decay, the exponential growth takes over and the length of the spring grows exponentially with time, as the lighter negative matter particle pulls away in contrary response to the increasing pull of the spring

$$r \to (r_0/2)e^{t/T} \tag{49}$$

Now that we have solved for the length of the spring, we can substitute Eq. (46) into Eqs. (21) and (22) for the motions of the two point particles to get

$$\ddot{x}_1 = \frac{kr}{M_+} = \frac{kr_0}{2M_+}(e^{t/T} + e^{-t/T}) \tag{50}$$

$$\ddot{x}_2 = \frac{kr}{M_-} = \frac{kr_0}{2M_-}(e^{t/T} + e^{-t/T}) \tag{51}$$

These two equations of motion have the well-known solutions

$$\begin{aligned} x_1 &= x_{10} + (kr_0 T^2/2M_+)(e^{t/T} + e^{-t/T} - 2) \\ &= x_{10} + (M_-/\Delta M)(r_0/2)(e^{t/T} + e^{-t/T} - 2) \end{aligned} \tag{52}$$

$$x_2 = x_{20} + (M_+/\Delta M)(r_0/2)(e^{t/T} + e^{-t/T} - 2) \tag{53}$$

where I have used Eq. (45) for the exponential growth time T.

Examining the behavior of Eqs. (52) and (53) with time, we see that both particles move off in the positive x direction, with the negative matter particle leading and the positive matter particle following. Since the magnitude of the mass of the positive matter particle is greater than that of the negative matter particle, the position x_2 of the negative matter particle grows faster than the position x_1 of the positive matter particle, so the spring stretches. The increased spring length increases the spring force, so that the system continues to accelerate at an exponentially increasing rate until something breaks.

The velocities of the two particles are obtained by taking the time derivative of Eqs. (52) and (53), giving

$$\dot{x}_1 = (M_- r_0/2\Delta M T)(e^{t/T} - e^{-t/T}) = (M_-/\Delta M)\dot{r} \tag{54}$$

$$\dot{x}_2 = (M_+/\Delta M)\dot{r} \tag{55}$$

To verify that this case satisfies the law of conservation of linear momentum, we calculate

$$\begin{aligned} \Sigma P &= M_1 \dot{x}_1 + M_2 \dot{x}_2 = (M_+ M_-/\Delta M)\dot{r} \\ &\quad - (M_- M_+/\Delta M)\dot{r} = 0 \end{aligned} \tag{56}$$

Since the positive linear momentum gained at any time by the positive matter particle is proportional to the magnitude of mass of the negative matter particle, and, similarly, for the negative linear momentum gained by the negative matter particle, the net momentum of the system remains zero.

To check on the law of conservation of energy, we calculate

$$\begin{aligned} 2\Sigma E &= M_1 \dot{x}_1^2 + M_2 \dot{x}_2^2 + kr^2 \\ &= \frac{M_+ M_-^2 - M_- M_+^2}{\Delta M^2} \dot{r}^2 + kr^2 \end{aligned} \tag{57}$$

If we use Eqs. (45–47), this can be converted to

$$\begin{aligned} 2\Sigma E &= -(k/4)r_0^2(e^{2t/T} - 2 + e^{-2t/T}) \\ &\quad + (k/4)r_0^2(e^{2t/T} + 2 + e^{-2t/T}) \\ &= kr_0^2 \end{aligned} \tag{58}$$

which is the initial energy in the spring at $t = 0$. Thus, the law of conservation of energy is not violated.

Note that if the magnitudes of the two masses are almost equal, then the motions of the two masses are similar, the spring remains nearly constant in length, and we approach the constant acceleration case.

Coupling of Objects Using Gravitational Forces

The analysis was then repeated, assuming that the force coupling between the two objects was due to the Newtonian gravitational force. Since the analysis of the simple Newtonian gravity case is instructive, but has not yet appeared in the published literature, I include a shortened version of it here. The situtation is that depicted in Fig. 4, where we have two particles, one of negative matter of mass $-M_-$ at position x_1, and one of positive matter of mass $+M_+$ at position x_2. As defined, the quantities M_+ and M_- are positive magnitudes for the two masses.

The two point particles in Fig. 4 have a separation distance r given by

$$r = x_2 - x_1 \tag{59}$$

where the positive sense of r is from x_1 to x_2.

The negative matter particle with active gravitational mass $-M_-$ causes a scalar gravitational potential ϕ_1 to appear at the position of the positive matter particle with a magnitude that is proportional to M_- and inversely proportional to r between the two particles

$$\phi_1 = +GM_-/r \tag{60}$$

where the potential is positive (repulsive), since the mass is negative.

The radial variations of this scalar gravitational potential, in turn, produce a vector gravitational acceleration field A_1 that is the negative of the gradient of the scalar potential

$$A_1 = -\text{grad}\phi_1 = +GM_-/r^2 \tag{61}$$

where as shown in Fig. 4, the direction of A_1 at the point x_2 is in the positive x direction since $-M_-$ is negative.

In a similar manner, the positive matter particle with active gravitational mass $+M_+$ produces a scalar gravitational potential ϕ_2 at the position x_1 with a magnitude given by

$$\phi_2 = +GM_+/r \tag{62}$$

where the positive sign is used because the sense of r is from the negative matter particle toward the positive matter particle. That potential, in turn, produces a vector gravitational acceleration field A_2 given by

$$A_2 = -\text{grad}\phi_2 = +GM_+/r^2 \tag{63}$$

where, as shown in Fig. 4, the direction of A_2 at the point x_1 is in the positive x direction since $+M_+$ is positive.

The negative passive gravitational mass $-M_-$ of the negative matter particle responds to A_2 of the positive matter particle to produce a force F_1 on the negative matter particle with a magnitude

$$F_1 = -M_-A_2 = -GM_-M_+/r^2 \tag{64}$$

where the direction of F_1 is in the opposite direction of A_2 because of the negative passive mass $-M_-$ of the negative matter particle.

In a similar manner, the positive passive mass $+M_+$ of the positive matter particle responds to A_1 of the negative matter particle to produce a force F_2 on the second particle with a magnitude

$$F_2 = +M_+A_1 = +GM_+M_-/r^2 \tag{65}$$

where the direction of F_2 is in the same direction as A_1, since the passive mass $+M_+$ is positive.

The F_1 on the negative matter particle causes the particle to move with an acceleration that is inversely proportional to the negative inertial mass $-M_-$ of the negative matter particle

$$\ddot{x}_1 = +F_1/M_- = +GM_+/r^2 \tag{66}$$

where the acceleration is in the positive x direction since the F_1 was in the negative x direction, and the negative inertial mass of the negative matter particle acts perversely to the applied force.

In a similar manner, F_2 on the second particle causes the particle to move with an acceleration that is inversely proportional to the positive inertial mass of that particle

$$\ddot{x}_2 = +F_2/M_+ = +GM_-/r^2 \tag{67}$$

where the acceleration is in the positive x direction, since the F_2 was in the positive x direction and the inertial mass is positive.

Note that if the passive gravitational and inertial masses of a particle are equal, even if they are negative, they drop out of the equations. The acceleration of a particle is independent of its mass and depends only on the mass of the gravitating body. This shows that a positive mass causes all other masses to accelerate toward it, whereas a negative mass causes all other masses to accelerate away from it.

Equations (66) and (67) are the two equations of motion that we wish to solve. The result when two objects are of equal but opposite mass is a unidirectional acceleration proportional to the gravity force between the two masses, as derived by Bondi.[2] I found that when two masses are not equal in magnitude, then, in addition to the unidirectional acceleration, a decrease or increase in the separation of the two objects is superimposed. Those readers interested in the detailed analysis can consult Ref. 7.

Coupling of Objects Using Electrostatic Forces

Although negative matter particles may not have convenient "handles" to hook springs to, and may not be dense enough to produce significant levels of gravitational forces, they may be massive enough to be useful and have an electrical charge. The electrical charge will give us a handle on the negative matter particle that can be used to "push" and "pull" the particle at a distance using electrostatic forces. The high force levels obtainable using electrostatic forces would produce high accelerations in a propulsion application. Electrostatic forces may also be used to build a computer-controlled, active

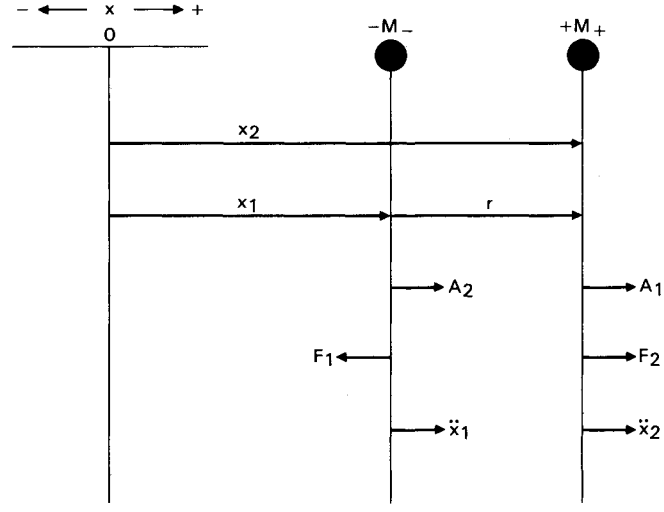


Fig. 4 Gravitational coupling of two opposite masses.

"trap" that can capture and collect large numbers of negative matter particles (provided they exist and are charged).

I also carried out a detailed analysis of the interaction of a positive and negative matter particle coupled by Coulomb electrostatic forces. I obtained the same results of unidirectional motion of the combined system at an acceleration proportional to the strength of the electrostatic force. Again, there was no violation of the laws of conservation of momentum and energy, even when the masses were of different magnitudes. You will have to take my word for it, as this paper is far too long already. My results are corroborated in a paper by Hoffmann,[8] in which he states, in one short paragraph, the results of his calculations of the interaction of two particles of equal and opposite masses through the Coulomb attraction when the charges are both equal and opposite. Unfortunately, Hoffmann's paper contains no mathematical equations for the charged particle case, preventing a more detailed comparison.

Other Publications on Negative Matter

In addition to the paper by Bondi[2] describing a negative mass chasing a positive mass, and the more detailed solution to the gravitational field equations by Bonner and Swaminarayan,[6] there have been a few other papers[9] that amplified on these previous papers, still finding no logical contradictions. Hoffmann[8] expanded the Bondi result to include charged particles of equal and opposite masses, but with no mathematical detail. Terletskii also mentions charged negative matter,[10] but again with no mathematical detail. A review of the more ancient philosophical discussions of negative mass can be found in a book by Jammer.[11]

Winterberg[12] has postulated that quarks are particles with both electric and magnetic charge *and* negative inertial mass. He also discussed using negative matter to reduce the net mass of a space vehicle to zero.

Inomata and Peak[13] have written a paper in which they reinterpret the principle of equivalence so that no repulsive solution is possible, making the phenomena of a negative mass chasing a positive mass implausible. I do not buy their "re-interpretation," but many would.

The most serious logical argument against the existence of negative matter is given by Martins,[14] who shows that if a hollow spherical shell of negative matter is surrounded by a hollow spherical shell of positive matter, there will be a gravitational force field between the two shells, but no gravitational forces outside the positive matter shell or inside the negative matter shell. However, the absolute gravitational *potential* and speed of light inside the negative matter shell would be higher than on the outside of the combination. Thus, if conditions were right, there could be a signal path that traversed through the inner region that would allow the propagation of signals from one point to another, faster than the speed of light outside the shells, allowing the construction of situations in which causality could be violated. The author himself points out that causality is violated by many solutions to the Einstein General Relativity field equations and that this is not sufficient reason to discard the possibilty of the existence of negative matter. Others would say violation of causality is enough to prove negative matter is logically impossible. These individuals must also discard the Einstein General Theory of Relativity if they are to be logically consistent. I prefer to believe in both.

Is Negative Matter Logically Forbidden?

In addition to the usual library literature research for this paper, I also wrote to a number of the authors of the references, sending along a preprint and asking if they knew of any logical reason why negative matter could not exist.

Weber published long ago this statement, "Nothing in either Newtonian or relativistic gravitational theory precludes the existence of negative mass."[15]

Dyson wrote, "I don't know any theorem that says negative mass is logically self-contradictory."[16]

Bondi wrote, "I am not aware of any logical exclusion of negative mass."[17]

Bonnor wrote, "I have [just] written a paper called 'Negative Mass in General Relativity.'[19] Why, I asked, is mass positive? My investigation fails to find a reason."[18]

Winterberg wrote to point out that if the Einstein Special Theory of Relativity is valid for all energies, then particles can decay from positive into negative energy states, and the vacuum would be unstable, which it is not. This would imply that negative matter particles, if they exist, probably would not be negative energy state versions of known positive matter particles, but would be completely different particles.

Where Is It?

If negative matter is not logically forbidden, then where is it? There do exist clues that may point to places where negative matter can be found.

One clue that there may be large amounts of negative matter in the universe can be found in papers discussing the huge voids found in large-scale three-dimensional "maps" of the universe.[21,22] As shown in Fig. 5, these "voids" or "bubbles" are 100 million lightyears across (our Milky Way galaxy is a mere 0.06 million lightyears across). The bubbles are sharply defined by large numbers of galaxies[22] that seem to lie on the surface of the bubbles. There are almost no galaxies in the voids,[23] and those that are found there are very unusual, characterized by strong, high-excitation emission spectra.[24] The present explanation is that these voids contain "failed" galaxies, or clouds of ordinary hydrogen gas.[25]

Another explanation for this "frothy" structure of the present-day universe is that it was formed with equal amounts of negative and positive matter particles. (This has the nice feature that the net mass of the universe is zero. The universe is the ultimate free lunch.) The voids are full of the negative matter particles trying to keep as far away from each other as possible, meanwhile pushing the positive mass particles to the surface of the voids where they attract each other to form galaxies, stars, planets, and us. One way to test this hypothesis is to use some of the available computer models for universes to see if an equal mixture of positive and negative matter would separate out into regions similar in size and shape to those observed.

Both Miller[26] and Baranov[27] have postulated cosmological models of universes containing particles of both positive and negative mass. Using his model, Baranov can explain the present experimental estimates for the cosmological constant and the Hubble red shift, as well as some anomalous effects seen in interacting galaxies.

Another clue to the possible existence of large amounts of negative matter is the "streaming" of large-scale structures at extremely high velocities. The local group, which includes our Milky Way galaxy, is streaming at 600 km/s relative to the microwave background rest-frame. This velocity is nearly impossible to account for within the current, cold dark-matter models of galaxy formation.[21] It remains to be seen whether a model assuming collective repulsion from "empty" regions containing large quantities of negative matter can reproduce the observed streaming.

In a more "local" type of anomalous dynamical behavior, it is well known that the orbital velocities of the stars in our galaxy (and other galaxies) remain high, even for stars far out from the central core mass of the galaxy.[21,28] The curves of the velocity of the stars vs their distance from the center of the galaxy are not consistent with the galactic mass distribution calculated from the star distribution. The present explanation is that a large fraction of the galaxy (as much as 90%) is made of invisible, "missing" matter that increases the effective mass of the galaxy over that calculated from the observable stars. My suggestion is that perhaps the peculiar rotation curves observed are caused by negative matter "pushing" from the outside, speeding up the orbits. This hypothesis needs a good

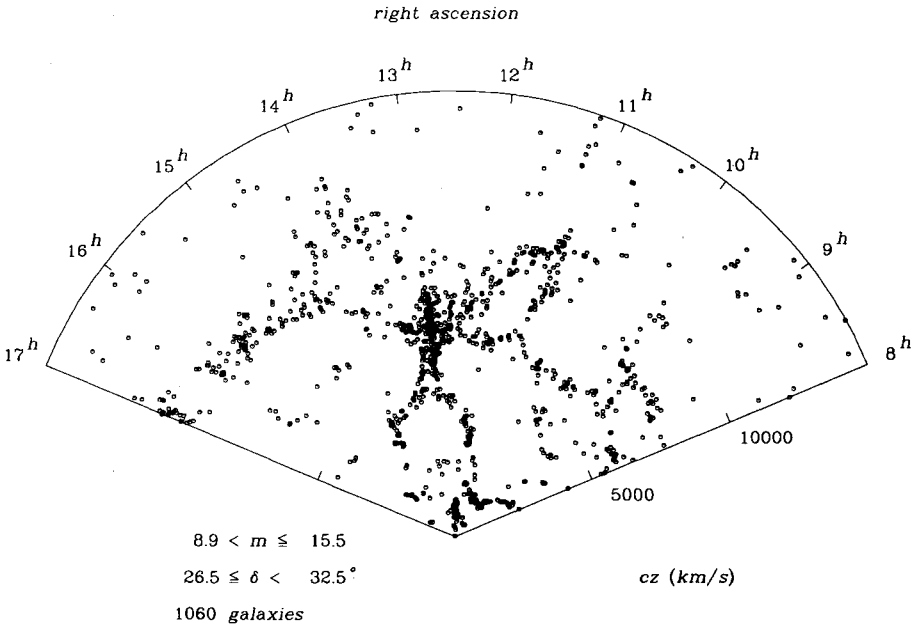


Fig. 5 A slice of the universe (from Ref. 22).

computer model and some testing to see if the numerical results of the model fit the observations.

Finally, if there are large numbers of negative matter particles in the universe, then positive masses, like our sun, will attract all other masses, including those negative matter particles. As Terletskii[29] has pointed out, when the negative matter particles strike the positive matter particles in the sun, they will continuously accelerate during the process, gaining negative kinetic energy and putting positive kinetic energy into the positive matter particles in the sun, heating it up. Terletskii argues that because of the absence of the catastrophic heating of the sun, there are either very few negative matter particles (read none) or their interaction cross section is low. It could be, instead, that there is heating of the sun by an influx of negative matter particles, and this is why the fusion neutrinos observed coming from the sun are only one-third that calculated if we assume all the energy in the sun is generated by gravitational contraction and thermonuclear fusion. Again, modeling is needed to see if the solar flux measurements are consistent with a local density of negative matter particles that is, in turn, consistent with large-scale universe models containing negative matter.

[Note added in proof: An alternate explanation for the lack of heating of the Sun and Earth by an influx of negative matter particles is to postulate that when a negative matter particle physically contacts a positive matter particle, a process called "nullification" takes place. Nullification would be a mutual particle destruction process similar to the annihilation process that occurs when an antimatter particle physically contacts a normal matter particle. In antimatter-normal matter annihilation, the two particles are destroyed and their combined rest mass is released as energy. In negative matter-positive matter nullification, the two particles would be destroyed, but since their combined rest mass is zero, *no* energy would be released. Such events would cause no heating and would be nearly impossible to detect.]

Conclusions

Using the Newtonian laws of mechanics and gravitation, I have examined the behavior of negative matter when interacting with positive matter through elastic, gravitational, and electrostatic forces. I find that when the negative matter object has a negative mass equal in magnitude to the positive mass of the positive matter object, both objects proceed to move off together in the same direction at a constant acceleration. This results in a method of propulsion that requires no internal or external source of propulsion energy and requires no reaction mass for expulsion in the opposite direction.

I have shown in exhaustive detail that this propulsion technique violates none of the Newtonian laws of conservation of linear momentum or total energy. Thus, Newton's Laws of Motion cannot be used to prove the nonexistence of negative matter and the infeasibility of negative matter propulsion. Although negative matter has not been observed to exist in the known universe, some suggestions are made where it might be found, and its presence detected. Once found, collecting the negative matter and using it for propulsion remain an exercise for the reader.

Personal Note to Skeptical Readers

Look, I know that negative matter propulsion is ridiculous and logically impossible. If so, it should be easy to prove it is logically impossible. But Bondi could not do it, Hoffmann could not do it, Terletskii could not do it, Bonner could not do it, and I could not do it. Can you?

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